95. $4\sin^2\theta = 1 + 4\cos\theta$ **96.** $8 - 12\sin^2\theta = 4\cos^2\theta$ **97.** $\sin(2\theta) = \sqrt{2}\cos\theta$

98. $1 + \sqrt{3}\cos\theta + \cos(2\theta) = 0$ **99.** $\sin\theta - \cos\theta = 1$ **100.** $\sin\theta - \sqrt{3}\cos\theta = 2$

In Problems 101–106, use a calculator to find an approximate value for each expression, rounded to two decimal places.

101. $\sin^{-1}0.7$ **102.** $\cos^{-1}\frac{4}{5}$ **103.** $\tan^{-1}(-2)$

104. $\cos^{-1}(-0.2)$ **105.** $\sec^{-1}3$ **106.** $\cot^{-1}(-4)$

In Problems 107–112, use a graphing utility to solve each equation on the interval $0 \le x \le 2\pi$. Approximate any solutions rounded to two decimal places.

107. $2x = 5\cos x$ **108.** $2x = 5\sin x$ **109.** $2\sin x + 3\cos x = 4x$

110. $3\cos x + x = \sin x$ **111.** $\sin x = \ln x$ **112.** $\sin x = e^{-x}$

113. Use a Half-angle formula to find the exact value of sin 15°. Then use a difference formula to find the exact value of sin 15°. Show that the answers found are the same.

114. If you are given the value of $\cos\theta$ and want the exact value of $\cos(2\theta)$, what form of the Double-angle Formula for $\cos(2\theta)$ is most efficient to use?

Chapter Test

In Problems 1–6, find the exact value of each expression. Express all angles in radians.

1. $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ **2.** $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ **3.** $\cos^{-1}\left(\sin\frac{11\pi}{6}\right)$

4. $\sin\left(\tan^{-1}\frac{7}{3}\right)$ **5.** $\cot\left(\csc^{-1}\sqrt{10}\right)$ **6.** $\sec\left(\cos^{-1}\left(-\frac{3}{4}\right)\right)$

In Problems 7–10, use a calculator to evaluate each espression. Express angles in radians.

7. $\sin^{-1}0.382$ **8.** $\sec^{-1}1.4$ **9.** $\tan^{-1}3$ **10.** $\cot^{-1}5$

In Problems 11–16 establish each identity.

11. $\frac{\csc\theta + \cot\theta}{\sec\theta + \tan\theta} = \frac{\sec\theta - \tan\theta}{\csc\theta - \cot\theta}$ **12.** $\sin\theta\tan\theta + \cos\theta = \sec\theta$ **13.** $\tan\theta + \cot\theta = 2\csc(2\theta)$

14. $\frac{\sin(\alpha+\beta)}{\tan\alpha + \tan\beta} = \cos\alpha\cos\beta$ **15.** $\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$ **16.** $\frac{\tan\theta - \cot\theta}{\tan\theta + \cot\theta} = 1 - 2\cos^2\theta$

In Problems 17–24 use sum, difference, product or half-angle formulas to find the exact value of each expression.

17. $\cos 15°$ **18.** $\tan 75°$ **19.** $\sin\left(\frac{1}{2}\cos^{-1}\frac{3}{5}\right)$

20. $\tan\left(2\sin^{-1}\frac{6}{11}\right)$ **21.** $\cos\left(\sin^{-1}\frac{2}{3} + \tan^{-1}\frac{3}{2}\right)$ **22.** $\sin 75°\cos 15°$

23. $\sin 75° + \sin 15°$ **24.** $\cos 65°\cos 20° + \sin 65°\sin 20°$

In Problems 25–29, solve each equation on $0 \le \theta < 2\pi$. → use cos(A-B) identity first

25. $4\sin^2\theta - 3 = 0$ **26.** $-3\cos\left(\frac{\pi}{2} - \theta\right) = \tan\theta$ **27.** $\cos^2\theta + 2\sin\theta\cos\theta - \sin^2\theta = 0$

28. $\sin(\theta + 1) = \cos\theta$ **29.** $4\sin^2\theta + 7\sin\theta = 2$

30. Stage 16 of the 2004 Tour de France was a time trial from Bourg d'Oisans to L'Alpe d'Huez. The average grade (slope as a percent) for most of the 15 km mountainous trek was 7.9%. What was the change in elevation from the beginning to the end of the route?

28 and 29 → graph in calculator using intersect or zero
* $\sin^2\theta$ MUST BE ENTERED $(\sin(x))^2$

Chapter Test *(page 514)*

1. $\frac{\pi}{6}$ **2.** $-\frac{\pi}{4}$ **3.** $\frac{2\pi}{3}$ **4.** $\frac{7\sqrt{58}}{58}$ **5.** 3 **6.** $-\frac{4}{3}$ **7.** ≈ 0.392 **8.** ≈ 0.775 **9.** ≈ 1.249 **10.** ≈ 0.197

11. $\frac{(\csc\theta + \cot\theta)}{(\sec\theta + \tan\theta)} = \frac{(\csc\theta + \cot\theta)}{(\sec\theta + \tan\theta)} \cdot \frac{(\csc\theta - \cot\theta)}{(\csc\theta - \cot\theta)} = \frac{(\csc^2\theta - \cot^2\theta)}{(\sec\theta + \tan\theta)(\csc\theta - \cot\theta)} = \frac{1}{(\sec\theta + \tan\theta)(\csc\theta - \cot\theta)}$

$= \frac{1}{(\sec\theta + \tan\theta)(\csc\theta - \cot\theta)} \cdot \frac{\sec\theta - \tan\theta}{\sec\theta - \tan\theta} = \frac{\sec\theta - \tan\theta}{(\sec^2\theta - \tan^2\theta)(\csc\theta - \cot\theta)} = \frac{\sec\theta - \tan\theta}{\csc\theta - \cot\theta}$

12. $\sin\theta\tan\theta + \cos\theta = \sin\theta \cdot \frac{\sin\theta}{\cos\theta} + \cos\theta = \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta$

13. $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta\cos\theta} + \frac{\cos^2\theta}{\sin\theta\cos\theta} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$

$= \frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin(2\theta)} = 2\csc(2\theta)$

14. $\frac{\sin(\alpha + \beta)}{\tan\alpha + \tan\beta} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta}} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta}}$

$= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{1} \cdot \frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta} = \cos\alpha\cos\beta$

15. $\sin(3\theta) = \sin(\theta + 2\theta) = \sin\theta\cos(2\theta) + \cos\theta\sin(2\theta) = \sin\theta \cdot (\cos^2\theta - \sin^2\theta) + \cos\theta \cdot 2\sin\theta\cos\theta = \sin\theta\cos^2\theta - \sin^3\theta + 2\sin\theta\cos^2\theta$

$= 3\sin\theta\cos^2\theta - \sin^3\theta = 3\sin\theta(1 - \sin^2\theta) - \sin^3\theta = 3\sin\theta - 3\sin^3\theta - \sin^3\theta = 3\sin\theta - 4\sin^3\theta$

16. $\frac{(\tan\theta - \cot\theta)}{(\tan\theta + \cot\theta)} = \frac{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} = \frac{\frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}}{\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}} = \frac{\sin^2\theta - \cos^2\theta}{\sin^2\theta + \cos^2\theta} = \frac{-\cos(2\theta)}{1} = -(2\cos^2\theta - 1) = 1 - 2\cos^2\theta$

17. $\frac{\sqrt{2}}{4}(\sqrt{3} + 1)$ **18.** $2 + \sqrt{3}$ **19.** $\frac{\sqrt{5}}{5}$ **20.** $\frac{12\sqrt{85}}{49}$ **21.** $\frac{2\sqrt{13}(\sqrt{5} - 3)}{39}$ **22.** $\frac{2 + \sqrt{3}}{4}$ **23.** $\frac{\sqrt{6}}{2}$ **24.** $\frac{\sqrt{2}}{2}$

25. $\left\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$ **26.** $\{0, 1.911, \pi, 4.373\}$ **27.** $\left\{\frac{3\pi}{8}, \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}\right\}$ **28.** $\{0.285, 3.427\}$ **29.** $\{0.253, 2.889\}$

30. The change in elevation during the time trial was 1.185 kilometers.